Find the co-ordinates of the vertices and foci, and the equations of the asymptotes of the hyperbola $x^2 - 9y^2 + 4x - 54y - 41 = 0$. **State clearly which co-ordinates are for which points.** **SCORE: ____ / 8 PTS**

$x^2+4x-9y^2-54y=41$

$x^2+4x+4-9(y^2+6y+9)=41+4-9\cdot 9$

$(x+2)^2-9(y+3)^2=-36$ (2)

$\frac{(y+3)^2}{4}-\frac{(x+2)^2}{36}=1$ (1)

2 ← (-2,-3)

$c^2=4+36=40$

$c=2\sqrt{10}$

CENTER $(-2,-3)$

VERTICES $(-2,-3\pm 2)=(-2,-1),(-2,-5)$ (1½)

FOCI $(-2,-3\pm 2\sqrt{10})$ (1½)

ASYMPTOTES SLOPE $=\pm\frac{\sqrt{4}}{\sqrt{36}}=\pm\frac{2}{6}=\pm\frac{1}{3}$

$y+3=\pm\frac{1}{3}(x+2)$ (1) (1)

Classify the graph of each equation as a circle, a parabola, an ellipse or a hyperbola. **SCORE: ____ / 2 PTS**

[a] $3x^2-5x+3y^2+4y+1=0$ CIRCLE (1)

[b] $7x^2+9x-5y^2+12y-2=0$ HYPERBOLA (1)

A point has polar coordinates $\left(14, \frac{2\pi}{9}\right)$. **SCORE: ____ / 2 PTS**

[a] Find another set of polar coordinates for the point, using a positive value of r.

$(14, \frac{2\pi}{9}\pm 2\pi)=(14, \frac{20\pi}{9})$ OR $(14, -\frac{16\pi}{9})$

(1) IF YOU GOT EITHER ANSWER

[b] Find another set of polar coordinates for the point, using a negative value of r.

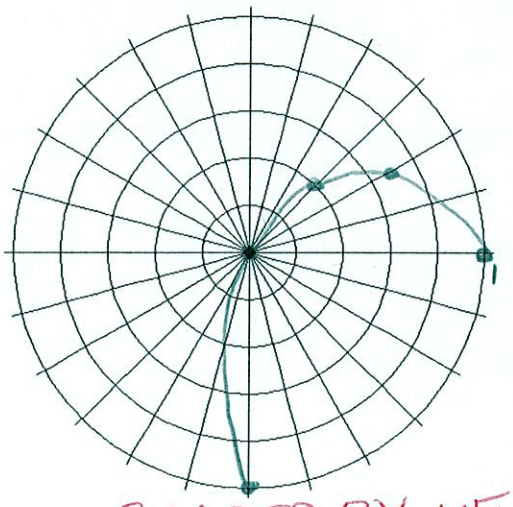
$(-14, \frac{2\pi}{9}\pm\pi)=(-14, \frac{11\pi}{9})$ OR $(-14, -\frac{7\pi}{9})$

(1) IF YOU GOT EITHER ANSWER

Plot the graph of $r = 2\cos\theta - 1$ for $0 \le \theta \le \frac{\pi}{2}$. You must list the polar coordinates for 5 points in that range. **SCORE: ____ / 8 PTS**
Use decimal approximations for irrational values of r.

POINTS ON GRAPH

(5)
- (1 , 0)
- (0.7 , $\frac{\pi}{6}$)
- (0.4 , $\frac{\pi}{4}$)
- (0 , $\frac{\pi}{3}$)
- (-1 , $\frac{\pi}{2}$)



GRADED BY ME

Convert the rectangular coordinates $(9, -3\sqrt{3})$ to polar coordinates. **SCORE: ____ / 4 PTS**

$r = \sqrt{81 + 27} = \sqrt{108} = 6\sqrt{3}$

$\sin\theta = \frac{-3\sqrt{3}}{6\sqrt{3}} = -\frac{1}{2}$

$\cos\theta = \frac{9}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{18} = \frac{\sqrt{3}}{2}$

$\theta = \frac{11\pi}{6}$

(1) IF IN CORRECT ORDER

$\left(6\sqrt{3}, \frac{11\pi}{6}\right)$

(1) (2)

Convert the polar equation $r^2 = 2 + \sin 2\theta$ to rectangular form. **SCORE: ____ / 6 PTS**

$r^2 = 2 + 2\sin\theta\cos\theta$ (2)

$r^2 = 2 + \frac{2xy}{r^2}$ (1½)

$r^4 = 2r^2 + 2xy$ (1) ← CAN BE WRITTEN AS $r^2(r^2 - 2) = 2xy$

$(x^2+y^2)^2 = 2(x^2+y^2) + 2xy$

$(x^2+y^2)(x^2+y^2-2) = 2xy$

(1½) IF YOU GOT EITHER ANSWER